

# From Inference to Physics\*

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## Abstract

Entropic dynamics, a program that aims at deriving the laws of physics from standard probabilistic and entropic rules for processing information, is developed further. We calculate the probability for an arbitrary path followed by a system as it moves from given initial to final states. For an appropriately chosen configuration space the path of maximum probability reproduces Newtonian dynamics.

## 1 Introduction

It is not unusual to hear that science consists in using information about the world for the purpose of predicting, modeling, and/or controlling phenomena of interest. If this vague image turns out to be even remotely accurate then we expect that the laws of science should reflect, at least to some extent, the methods for manipulating information. Here we wish to entertain a far more radical hypothesis: perhaps *the laws of physics are nothing but rules of inference*. In this view the laws of physics are not laws of nature but are merely the rules we follow when processing the information that happens to be relevant to the physical problem at hand. The evidence supporting this notion is already quite considerable: most of the formal structure of statistical mechanics [1] and of quantum theory (see e.g. [2]) can be derived as examples of inference.

The basic difficulty is that the available information is usually incomplete and one must learn to handle uncertainty. This requires addressing three problems; the first two have been satisfactorily solved, the third one has not. First, one must represent one's partial state of knowledge as a web of interconnected beliefs with no internal inconsistencies; the tools to do it are probabilities [3, 4]. Second, when new information becomes available the beliefs must be correspondingly updated. The instrument for updating is relative entropy and the resulting procedure—the ME method—is the only candidate that can claim universal applicability. The ME method is based on the recognition that prior information

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is valuable and should not be revised except when demanded by new evidence; it can handle arbitrary priors and arbitrary constraints; it includes MaxEnt and Bayes' rule as special cases; and it provides a quantitative assessment of the extent that distributions that deviate from the entropy maximum are ruled out. (See e.g. [5, 6].)

The third problem is trickier. When we say that “the laws of physics are not laws of nature” we do not mean that physics can be derived without any input from nature; quite the opposite. The statement “physics is inference” comes with considerable fine print. It implicitly assumes that one is doing inference about the “right things” on the basis of the “right information.” The third and so far unsolved problem is that of identifying the questions that are interesting and the information that is relevant about a particular physical situation—this is where the connection to nature lies. The current approaches cannot be called a method—ultimately there is no scientific “method.” We have learned from experience—a euphemism for trial and error, mostly error—which pieces of information happen to work well in each specific situation. Recent results, however, in model selection [7] and in the development of a quantitative theory of inquiry and of relevance [8] represent considerable progress and point the way towards more systematic approaches.

In any case, once the relevant information has been identified, if the laws of physics are merely rules of inference, then we should be able to derive them. Our main concern is to derive laws of dynamics and the challenge—of course—is to avoid assuming the very laws of motion that we set out to derive. The formalism, which we refer to as *entropic dynamics* [9, 10], is of general applicability but to be specific we focus on the example of particle mechanics.

In a previous paper [11] we derived Newtonian mechanics without assuming a principle of least action, or concepts of force, or momentum, or mass, and not even the notion of an absolute Newtonian time. None of these familiar concepts are part of the input to the theory—they are all derived. As described in [11] the crucial step was the selection of a suitable statistical model for the configuration space of a system of particles which amounts to specifying both the subject matter and the relevant background information.

The objective of the present paper is to develop the formalism of entropic dynamics further. We address the same dynamically interesting question: Given an initial and a final state, what trajectory will the system follow? In [11] we had calculated the path of maximum probability and we showed that it corresponds to Newtonian dynamics. But the available information does not single out a unique path; here—and this is our main result—we calculate the probability for any arbitrary path between the given initial and final states. As a first application we verify that indeed the most probable path reproduces our earlier result. A more detailed study of fluctuations and diffusion about the Newtonian path will, however, be left for a future publication.

We conclude with brief remarks about the asymmetry between past and future as seen from the unfamiliar perspective of entropic dynamics, and about a possible connection between this work and Nelson's derivation of quantum mechanics as a peculiar kind of diffusion process [12].

## 2 Physical space and configuration space

Consider one particle (or many) living in our familiar “physical” space (whatever this might ultimately mean). There is a useful distinction to be drawn between this physical space  $\mathcal{Y}$  and the space of states or configuration space  $\mathcal{X}$ . For simplicity we will assume that physical space  $\mathcal{Y}$  is flat and three dimensional; its geometry is given by the Euclidean metric  $ds^2 = \delta_{ab} dy^a dy^b$ —generalizations are straightforward. The configuration space  $\mathcal{X}$  for a single particle will also be assumed to be three dimensional but it need not be flat. The interesting dynamics will arise from its curvature. The main additional ingredient is that there is an irreducible uncertainty in the location of the particle. Thus, when we say that the particle is at the point  $x \in \mathcal{X}$  what we mean is that its “physical”, “true” position  $y \in \mathcal{Y}$  is somewhere in the vicinity of  $x$ . This leads us to associate a probability distribution  $p(y|x)$  to each point  $x$  and the space  $\mathcal{X}$  is thus transformed into a statistical manifold: a point  $x$  is not a structureless dot but a fuzzy probability distribution. The origin of these uncertainties is, at this point, left unspecified.

In [11] we adopted a Gaussian model,

$$p(y|x) = \frac{\gamma^{1/2}(x)}{(2\pi)^{3/2}} \exp \left[ -\frac{1}{2} \gamma_{ab}(x) (y^a - x^a)(y^b - x^b) \right], \quad (1)$$

where  $\gamma = \det \gamma_{ab}$ . It incorporates the physically relevant information of an estimate of the particle position,  $\langle y^a \rangle = x^a$ , and of its small uncertainty as given by the covariance matrix,

$$\tilde{\gamma}^{ab} = \langle (y^a - x^a)(y^b - x^b) \rangle, \quad (2)$$

which is the inverse of  $\gamma_{ab}$ ,  $\tilde{\gamma}^{ab} \gamma_{bc} = \delta_c^a$ . The choice of Gaussians is physically plausible but not strictly necessary. We are trying to predict behavior at macro-scales in terms of assumptions we make (that is, information we assume) about what is going on at some intermediate meso-scales which themselves are the result of happenings at still shorter micro-scales about which we know absolutely nothing. If the fuzziness in position that we postulate at the meso-scale is the result of many unknown microscopic influences going on at a much smaller micro-scale then general arguments such as the central limit theorem lead us to expect Gaussians as the plausible mesoscopic distributions for a very wide variety of microscopic conditions.

To conclude the specification of the model we further impose that the Gaussians be spherically symmetric<sup>1</sup> with a small but non-uniform variance  $\sigma^2(x)$  conveniently expressed in terms of a small constant  $\sigma_0^2$  modulated by a (positive) scalar field  $\Phi(x)$ ,

$$\gamma_{ab}(x) = \frac{1}{\sigma^2(x)} \delta_{ab} = \frac{\Phi(x)}{\sigma_0^2} \delta_{ab}. \quad (3)$$

The next feature is automatic, it requires no further assumptions: the configuration space  $\mathcal{X}$ , when viewed as a statistical manifold, inherits a geometry

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<sup>1</sup>This corresponds to the Newtonian assumption that space is locally isotropic.

from the distributions  $p(y|x)$ . The distance between two neighboring distributions  $p(y|x)$  and  $p(y|x+dx)$  is the *unique* measure of the extent that one distribution can be statistically distinguished from the other—*distinguishability is distance* (and possibly vice-versa, but that is a story for another paper [10]). It is given by the information metric of Fisher and Rao [13, 6],

$$d\ell^2 = g_{ab} dx^a dx^b \quad \text{with} \quad g_{ab} = \int dy p(y|x) \frac{\partial \log p(y|x)}{\partial x^a} \frac{\partial \log p(y|x)}{\partial x^b} . \quad (4)$$

The corresponding volume element is  $dv = g^{1/2}(x) d^3x$  where  $g = \det g_{ab}$ . Substituting (1) and (3) into (4) we obtain the information metric for the manifold of spherically symmetric Gaussians,

$$g_{ab}(x) = \frac{1}{\sigma^2(x)} (\delta_{ab} + 6\partial_a \sigma \partial_b \sigma) \approx \frac{\Phi(x)}{\sigma_0^2} \delta_{ab} = \gamma_{ab}(x) , \quad (5)$$

provided  $\sigma_0$  is sufficiently small.

### 3 The probability of a path

The fact that the state  $x$  of the particle might be unknown is described by a distribution  $P(x)$ . The path of a particle is an ordered sequence of  $N+1$  positions  $\{x_0 \dots x_N\}$ . We want to calculate the probability

$$P(x_1 \dots x_{N-1} | x_0 x_N) dx_1 \dots dx_{N-1} \quad (6)$$

that the path passes through small volume elements  $dx_n$  at the intermediate points  $x_1 \dots x_{N-1}$ . Since

$$P(x_1 \dots x_{N-1} | x_0 x_N) = \frac{P(x_1 \dots x_{N-1} x_N | x_0)}{P(x_N | x_0)} , \quad (7)$$

our immediate interest will be to assign a probability  $P(x_1 \dots x_N | x_0)$  of the ordered path  $\{x_1 \dots x_N\}$  starting at  $x_0$ .

Note that an external time has not been introduced. It is true that the path is ordered so that *along a given path* the point  $x_n$  reached after  $n$  steps could be construed to occur *later* than the point reached at the *previous* step,  $x_{n-1}$ . But most important elements implicit in the notion of time are conspicuously absent. For example, we still have no way to order temporally the point  $x_n$  reached along one path with the point  $x'_n$  reached along a different path. Statements to the effect that one occurs earlier or later or simultaneously with the other are, at this point, completely meaningless. We have not introduced a notion of *simultaneity* and therefore we do not have a notion of an *instant of time*. Furthermore we do not have a notion of *duration* either; we have not introduced a way to compare or measure intervals for the successive steps. The statement that a certain step took, say, twice as long as the previous step is, at this point, meaningless.

Without the notions of instant or of interval we do not have time. An important part of the program of deriving physics from inference consists in understanding how and where these temporal concepts arise.

### 3.1 The single-step probability

To warm up we first calculate the probability  $P(x_1|x_0)$  to reach  $x_1$  in a single step. Since we are ignorant not only about the true position  $y_1$  but also about the configuration space position  $x_1$  the relevant distribution of interest is the joint distribution  $P_J(x_1 y_1|x_0)$ . We shall choose the distribution  $P_J(x_1 y_1|x_0)$  using the ME method, that is, by maximizing the single-step entropy [6]

$$\mathcal{S}_1[P_J, P_J'] = - \int dx_1 dy_1 P_J(x_1 y_1|x_0) \log \frac{P_J(x_1 y_1|x_0)}{P_J'(x_1 y_1|x_0)} . \quad (8)$$

**The prior:**  $P_J'(x_1 y_1|x_0)$  represents partial knowledge about the variables  $x_1$  and  $y_1$  *before* we incorporate any information in the form of constraints. Let

$$P_J'(x_1 y_1|x_0) = P'(x_1|x_0)P'(y_1|x_0 x_1) , \quad (9)$$

and focus first on  $P'(x_1|x_0)$ . At this point it is not yet known how  $x_1$  is related to  $x_0$  or to  $y_1$ . We do know that  $x_1 \in \mathcal{X}$  labels some probability distribution  $p(y|x_1)$  in  $\mathcal{X}$ , eqs.(1, 3), but we do not yet know that it is the distribution of  $y_1$ ,  $p(y_1|x_1)$ . Thus, we are maximally ignorant about  $x_1$  and, accordingly, we choose a uniform distribution  $P'(x_1|x_0) \propto g^{1/2}(x_1)$ . For the second factor  $P'(y_1|x_0 x_1)$  we argue that the variables  $y_1$  are meant to represent the actual (uncertain) coordinates of a particle; we assume that in the absence of any information to the contrary the distribution of  $y_1$  remains unchanged from the previous step,  $P'(y_1|x_0 x_1) = p(y_1|x_0)$ . Thus, the joint prior  $P'$  is

$$P_J'(x_1 y_1|x_0) \propto g^{1/2}(x_1)p(y_1|x_0) . \quad (10)$$

**The constraint:** Next we incorporate the piece of information that establishes the relation between  $x_1$  and  $y_1$ . This is the constraint that demands updating from the prior to the posterior. The posterior  $P_J(x_1 y_1|x_0)$  belongs to the family of distributions

$$P_J(x_1 y_1|x_0) = P(x_1|x_0)P(y_1|x_1 x_0) \quad (11)$$

where  $P(x_1|x_0)$  is arbitrary and the second factor is constrained to be of the form  $P(y_1|x_0 x_1) = p(y_1|x_1)$ .<sup>2</sup>

Substituting (10) and (11) into (8) and rearranging gives

$$\mathcal{S}_1[P_J, P_J'] = - \int dx_1 P(x_1|x_0) \left[ \log \frac{P(x_1|x_0)}{g^{1/2}(x_1)} - S(x_1, x_0) \right] , \quad (12)$$

where

$$S(x_1, x_0) = - \int dy_1 p(y_1|x_1) \log \frac{p(y_1|x_1)}{p(y_1|x_0)} . \quad (13)$$

To determine  $P(x_1|x_0)$  maximize eq.(12) subject to normalization. The first term in eq.(12) makes  $x_1$  as random as possible; by itself it would lead to a

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<sup>2</sup>Incidentally, this is an example of an application of the ME method where the constraints are not in the form of expected values; these are not “linear” constraints.

uniform distribution  $P(x_1|x_0) \propto g^{1/2}(x_1)$ . The second term in eq.(12) brings  $x_1$  as close as possible to  $x_0$ ; it would make  $P(x_1|x_0) \propto \delta(x_1 - x_0)$  and push  $S(x_1, x_0)$  towards its maximum value,  $S(x_0, x_0) = 0$ . The compromise between these two opposing tendencies is

$$P_1(x_1|x_0) = \frac{1}{z(x_0)} g^{1/2}(x_1) e^{S(x_1, x_0)} , \quad (14)$$

where  $z(x_0)$  is an appropriate normalization constant.

The probability (14) represents a *discontinuous* jump from  $x_0$  to  $x_1$  in a single step. No information has been imposed to the effect that the particle “moves” from  $x_0$  to  $x_1$  along a continuous trajectory. This is done next by assuming that the continuous trajectory can be approximated by a sequence of  $N$  steps where  $N$  is large.

### 3.2 The $N$ -step probability

To assign the probability  $P(x_1 \dots x_N | x_0)$  for a path we focus on the joint distribution  $P_J(x_1 y_1 \dots x_N y_N | x_0)$  and choose the distribution that maximizes

$$\mathcal{S}_N[P_J, P'_J] = - \int \left( \prod_{n=1}^N dx_n dy_n \right) P_J \log \frac{P_J}{P'_J} . \quad (15)$$

**The prior:** To assign  $P'_J(x_1 y_1 \dots x_N y_N | x_0)$  consider the path in the spaces of  $xs$  and  $ys$  separately ( $x \in \mathcal{X}$  and  $y \in \mathcal{Y}$ ).

$$P'_J(x_1 y_1 \dots x_N y_N | x_0) = P'(x_1 \dots x_N | x_0) P'(y_1 \dots y_N | x_0 x_1 \dots x_N) . \quad (16)$$

The first factor is the prior probability of a path in the space  $\mathcal{X}^N$ . To the extent that we know nothing about the relation between successive  $xs$  we choose a uniform distribution in the space of paths,

$$P'(x_1 \dots x_N | x_0) \propto \prod_{n=1}^N g^{1/2}(x_n) . \quad (17)$$

The second factor is the prior probability of a path in the space  $\mathcal{Y}^N$ . We assume that in the absence of any information to the contrary the distribution of the  $n$ -th step  $y_n$  retains memory only of the immediately preceding  $x_{n-1}$ ,

$$P'(y_1 \dots y_N | x_0 x_1 \dots x_N) = \prod_{n=1}^N p(y_n | x_{n-1}) . \quad (18)$$

(This is not quite a Markov process; the distribution of  $y_n$  does not retain memory of the immediately preceding  $y_{n-1} \in \mathcal{Y}$ , only of  $x_{n-1} \in \mathcal{X}$ .)

**The constraint:** Next we impose the information that relates  $y_n$  to its corresponding  $x_n$ . The posterior

$$P_J(x_1 y_1 \dots x_N y_N | x_0) = P(x_1 \dots x_N | x_0) P(y_1 \dots y_N | x_0 x_1 \dots x_N) \quad (19)$$

is constrained to belong to the family of distributions such that

$$P(y_1 \dots y_N | x_0 x_1 \dots x_N) = \prod_{n=1}^N p(y_n | x_n) . \quad (20)$$

Substituting  $P_J$  and  $P'_J$  into eq.(15) and rearranging gives

$$\begin{aligned} \mathcal{S}_N[P_J, P'_J] = & -\int \left( \prod_{n=1}^N dx_n \right) P(x_1 \dots x_N | x_0) \log \frac{P(x_1 \dots x_N | x_0)}{\prod_{n=1}^N g^{1/2}(x_n)} \\ & + \int \left( \prod_{n=1}^N dx_n \right) P(x_1 \dots x_N | x_0) \sum_{n=1}^N S(x_n, x_{n-1}) \end{aligned} \quad (21)$$

where

$$S(x_n, x_{n-1}) = -\int dy_n p(y_n | x_n) \log \frac{p(y_n | x_n)}{p(y_n | x_{n-1})} \quad (22)$$

As before the two integrals in eq.(21) represent opposing tendencies. The first integral seeks to make  $P(x_1 \dots x_N | x_0)$  as random as possible with  $x_n$  completely uncorrelated to  $x_{n-1}$ . The second integral introduces strong correlations; it brings  $x_n$  as close as possible to the preceding  $x_{n-1}$ .

**The main result:** Varying  $P(x_1 \dots x_N | x_0)$  to maximize eq.(21) subject to normalization gives the probability density for a path starting at the initial position  $x_0$ ,

$$P_N(x_1 \dots x_N | x_0) = \frac{1}{Z(x_0)} \left[ \prod_{n=1}^N g^{1/2}(x_n) \right] \exp \left[ \sum_{n=1}^N S(x_n, x_{n-1}) \right] \quad (23)$$

where  $Z(x_0)$  is the appropriate normalization constant.

The probability density for the  $N$ -step path between given initial and final positions  $x_0$  and  $x_N$  is given by (7) where  $P_N(x_N | x_0)$  is obtained from (23),

$$P_N(x_N | x_0) = \frac{g^{1/2}(x_N)}{Z(x_0)} \int \left[ \prod_{n=1}^{N-1} dx_n g^{1/2}(x_n) \right] \exp \left[ \sum_{n=1}^N S(x_n, x_{n-1}) \right] . \quad (24)$$

Substituting back into (7) gives the desired answer

$$P_N(x_1 \dots x_{N-1} | x_0 x_N) = \frac{1}{Z(x_0, x_N)} \left[ \prod_{n=1}^{N-1} g^{1/2}(x_n) \right] \exp \left[ \sum_{n=1}^N S(x_n, x_{n-1}) \right] , \quad (25)$$

where  $Z(x_0, x_N)$  is the appropriate normalization. Equations (23) and (25) are the main results of this paper.

## 4 The most probable path

We restrict our analysis of eq.(25) to calculating the most probable path from the initial position  $x_0$  to the final  $x_N$ . For fixed volume elements,  $dV_n = g^{1/2}(x_n) dx_n = dV$ , the path of maximum probability is that which maximizes

$$A(x_1 \dots x_{N-1} | x_0 x_N) = \sum_{n=1}^N S(x_n, x_{n-1}) , \quad (26)$$

where  $x_0$  and  $x_N$  are fixed. The maximum probability path is the polygonal path that brings the successive  $x$ s as “close” to each other as possible. For large  $N$  we expect this to be the shortest path between the given end points and, as shown below, this is indeed the case. The variation of  $A$  is

$$\begin{aligned}\delta A &= \sum_{n=1}^{N-1} \frac{\partial}{\partial x_n^c} \left[ \sum_{m=1}^N S(x_m, x_{m-1}) \right] \delta x_n^c \\ &= \sum_{n=1}^{N-1} \frac{\partial}{\partial x_n^c} [S(x_n, x_{n-1}) + S(x_{n+1}, x_n)] \delta x_n^c .\end{aligned}\quad (27)$$

For large  $N$  we assume that successive  $x$ s along the path are sufficiently close together that we can approximate

$$S(x_n, x_{n-1}) = -\frac{1}{2} d\ell_{n,n-1}^2 = -\frac{1}{2} g_{ab}(x_{n-1})(x_n^a - x_{n-1}^a)(x_n^b - x_{n-1}^b) . \quad (28)$$

Next, introduce a parameter  $\lambda$  along the trajectory,  $x = x(\lambda)$ . The corresponding velocities  $\dot{x}$  are

$$\dot{x}_{n+1/2} \stackrel{\text{def}}{=} \frac{x_{n+1} - x_n}{\Delta\lambda}, \quad \dot{x}_{n-1/2} \stackrel{\text{def}}{=} \frac{x_n - x_{n-1}}{\Delta\lambda} \quad \text{and} \quad \dot{x}_n \stackrel{\text{def}}{=} \frac{x_{n+1} - x_{n-1}}{2\Delta\lambda} . \quad (29)$$

Expand,<sup>3</sup>

$$g_{ac}(x_{n-1}) = g_{ac}(x_n) - g_{ac,d}(x_n) \dot{x}_{n-1/2}^d \Delta\lambda + \dots, \quad (30)$$

and rearrange to get

$$\begin{aligned}\delta A &= - \sum_{n=1}^{N-1} \Delta\lambda^2 \left[ \frac{1}{2} g_{ab,c}(x_n) \dot{x}_{n+1/2}^a \dot{x}_{n+1/2}^b \right. \\ &\quad \left. + g_{ac}(x_n) \frac{\dot{x}_{n-1/2}^a - \dot{x}_{n+1/2}^a}{\Delta\lambda} - g_{ac,d}(x_n) \dot{x}_{n-1/2}^d \dot{x}_{n-1/2}^a \right] ,\end{aligned}\quad (31)$$

where we recognize the acceleration

$$\ddot{x}_n \stackrel{\text{def}}{=} \frac{\dot{x}_{n+1/2} - \dot{x}_{n-1/2}}{\Delta\lambda} = \frac{x_{n+1} - 2x_n + x_{n-1}}{(\Delta\lambda)^2} . \quad (32)$$

Substituting gives

$$\begin{aligned}\delta A &= \Delta\lambda^2 \sum_{n=1}^{N-1} \left[ g_{ac}(x_n) \ddot{x}_n^a + g_{ac,d} \left( \dot{x}_n^a - \ddot{x}_n^a \frac{\Delta\lambda}{2} \right) \left( \dot{x}_n^d - \ddot{x}_n^d \frac{\Delta\lambda}{2} \right) \right. \\ &\quad \left. - \frac{1}{2} g_{ab,c} \left( \dot{x}_n^a + \ddot{x}_n^a \frac{\Delta\lambda}{2} \right) \left( \dot{x}_n^b + \ddot{x}_n^b \frac{\Delta\lambda}{2} \right) \right] \delta x_n^c .\end{aligned}\quad (33)$$

If the distribution of points along the trajectory is sufficiently dense,  $\Delta\lambda \rightarrow 0$ , the leading term is

$$\delta A = \Delta\lambda^2 \sum_{n=1}^{N-1} \left[ g_{ac}(x_n) \ddot{x}_n^a + \frac{1}{2} (g_{ca,b} + g_{cb,a} - g_{ab,c}) \dot{x}_n^a \dot{x}_n^b \right] \delta x_n^c , \quad (34)$$

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<sup>3</sup>We use the standard notation  $g_{ac,d} = \partial g_{ac} / \partial x^d$ .



or

$$\delta A = \Delta \lambda^2 \sum_{n=1}^{N-1} g_{ad}(x_n) [\ddot{x}_n^a + \Gamma_{bc}^a \dot{x}_n^b \dot{x}_n^c] \delta x_n^d , \quad (35)$$

where  $\Gamma_{ab}^c$  are Christoffel symbols,

$$\Gamma_{ab}^c = \frac{1}{2} g^{cd} (g_{da,b} + g_{db,a} - g_{ab,d}) . \quad (36)$$

Setting  $\delta A = 0$  for arbitrary variations  $\delta x_n^d$  leads to the geodesic equation,

$$\ddot{x}_n^a + \Gamma_{bc}^a \dot{x}_n^b \dot{x}_n^c = 0 , \quad (37)$$

Incidentally,  $\lambda$  turns out to be an affine parameter, that is, up to an unimportant scale factor it measures the length along the path,

$$d\lambda^2 = C g_{ab} dx^a dx^b . \quad (38)$$

**Conclusion:** The most probable continuous path between two given end points is the geodesic that joins them.

**Remark:** It is interesting that although the ME inference implicitly assumed a directionality from the initial  $x_0$  to the final  $x_N$  through the prior for  $y_n$  which establishes a connection with the “previous” instant,  $p(y_n|x_{n-1})$ , in the continuum limit the sense of direction is lost. The most probable trajectory is fully reversible.

The treatment above is general; it is valid for dynamics on any statistical manifold. Now we restrict ourselves to the manifold of spherically symmetric Gaussians defined by (1, 3). The parametrization in terms of  $\lambda$  is convenient but completely arbitrary. Let us instead introduce a new non-affine “time” parameter  $t = t(\lambda)$  defined by

$$dt = \frac{d\lambda}{2^{1/2}\Phi} \quad \text{or} \quad T_t \stackrel{\text{def}}{=} \frac{1}{2\sigma_0^2} \delta_{ab} \frac{dx^a}{dt} \frac{dx^b}{dt} = \Phi \quad (39)$$

then the geodesic equation becomes

$$\frac{d^2 x^c}{dt^2} + \Gamma_{ab}^c \frac{dx^a}{dt} \frac{dx^b}{dt} = - \frac{d^2 t / d\lambda^2}{(dt/d\lambda)^2} \frac{dx^c}{dt} , \quad (40)$$

and using (36)

$$\Gamma_{ab}^c = \frac{1}{2\Phi} (\partial_a \Phi \delta_b^c + \partial_b \Phi \delta_a^c - \partial_c \Phi \delta_{ab}) , \quad (41)$$

we get

$$\frac{d^2 x^a}{dt^2} = \frac{\partial_a \Phi}{\Phi} \frac{1}{2} \delta_{bc} \frac{dx^b}{dt} \frac{dx^c}{dt} , \quad (42)$$

which, using  $T_t = \Phi$  from eq.(39), gives

$$\frac{1}{\sigma_0^2} \frac{d^2 x^a}{dt^2} = \partial_a \Phi \frac{T_t}{\Phi} = \partial_a \Phi . \quad (43)$$

This is Newton’s equation. To make it explicit just change notation and call

$$\frac{1}{\sigma_0^2} \stackrel{\text{def}}{=} m \quad \text{and} \quad \Phi(x) \stackrel{\text{def}}{=} E - V(x) \quad (44)$$

where  $E$  is a constant. The result is Newton’s  $F = ma$  and energy conservation,

$$m \frac{d^2 x^a}{dt^2} = - \frac{\partial V(x)}{\partial x^a} \quad \text{and} \quad \frac{m}{2} \delta_{ab} \frac{dx^a}{dt} \frac{dx^b}{dt} + V(x) = E . \quad (45)$$

**Conclusion:** We have reproduced the results obtained in [11]. The Newtonian mass  $m$  and force  $F^a = -\partial_a V$  are “explained” in terms of position uncertainties; the uniform uncertainty  $\sigma_0$  explains mass, while the modulating field  $\Phi(x)$  explains forces.

The extension to more particles interacting among themselves is straightforward—see [11]. Further analysis will be pursued elsewhere. Here we only mention that a most remarkable feature of the time  $t$  selected according to (39) is that *isolated* subsystems all keep the same common time which confirms  $t$  as the universal Newtonian time. Thus, the advantage of the Newtonian time goes beyond the fact that it simplifies the equations of motion. It is the only choice of time such that isolated clocks will keep synchronized.

## 5 Final remarks

We conclude with two comments. The first concerns the arrow of time, an interesting puzzle that has plagued physics ever since Boltzmann [14]. The problem is that the laws of physics are symmetric under time reversal—forget, for the moment, the tiny T violations in K-meson decay—but everything else in nature seems to indicate a clear asymmetry between the past and the future. How can we derive an arrow of time from underlying laws of nature that are symmetric? The short answer is: we can’t.

In a few brief lines we cannot do full justice to this problem but we can hint that entropic dynamics offers a promising new way to address it. We note, first, that entropic dynamics does not assume any underlying laws of nature—whether they be symmetric or not. And second, that information about the past is treated differently from information about the future. Entropic dynamics does not attempt to explain the asymmetry between past and future. The asymmetry is accepted as prior information. It is the known but unproven truth that provides the foundation from which all sorts of other inferences will be derived. From the point of view of entropic dynamics the problem is not to explain the arrow of time, but rather to explain the reversibility of the laws of physics. And in this endeavor entropic dynamics succeeds. Laws of physics such as  $\vec{F} = m\vec{a}$  were derived to be time reversible despite the fact that the entropic argument clearly stipulates an arrow of time. More generally, we showed that the probability of any *continuous* path is independent of the direction in which it is traversed. (Incidentally, if the paths were not continuous but composed of small discrete steps then the predictions would include tiny T violations.)

The second comment concerns the “physical” origin of the position uncertainties, an important issue about which we have remained silent. The fact that particle masses are a manifestation of these uncertainties,  $\sigma_0^2 \propto 1/m$ , might be a clue. Among the various approaches to quantum theory the version developed by Nelson, and known as stochastic mechanics [12], is particularly attractive because it derives quantum theory from the hypothesis that particles in empty space are subject to a peculiar Brownian motion characterized by position fluctuations such that  $\sigma^2 \propto \hbar/m$ . It is difficult to avoid the conclusion that the uncertainties underlying entropic dynamics might be explained by quantum effects. However, while this is a very tempting possibility, an even more interesting and radical conjecture is that the explanatory arrow runs in the opposite direction. The radical conjecture would be that the same entropic dynamics that already explains mass, and interactions, and Newton’s mechanics, might also—and with no further assumptions—explain quantum mechanics as well. Perhaps *physics is nothing but inference* after all.

## References

- [1] E. T. Jaynes: Phys. Rev. **106**, 620 and **108**, 171 (1957); *E. T. Jaynes: Papers on Probability, Statistics and Statistical Physics*, ed. by R. D. Rosenkrantz (Reidel, Dordrecht, 1983).
- [2] A. Caticha: Phys. Lett. **A244**, 13 (1998); Phys. Rev. **A57**, 1572 (1998); Found. Phys. **30**, 227 (2000) (arXiv.org/abs/quant-ph/9810074); “From Objective Amplitudes to Bayesian Probabilities” in *Foundations of Probability and Physics-4*, ed. by G. Adenier, C. Fuchs, and A. Khrennikov, AIP Conf. Proc. Vol. 889, 62 (2007) (arXiv.org/abs/quant-ph/0610076).
- [3] R. T. Cox, ‘Probability, Frequency and Reasonable Expectation’, Am. J. Phys. **14**, 1 (1946); *The Algebra of Probable Inference* (Johns Hopkins, Baltimore, 1961).
- [4] E. T. Jaynes, *Probability Theory: The Logic of Science*, ed. by L. Bretthorst (Cambridge University Press, Cambridge, 2003).
- [5] A. Caticha, “Information and Entropy,” in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by K. Knuth *et al.*, AIP Conf. Proc. **954**, 11 (2007) (arXiv.org/abs/0710.1068).
- [6] A. Caticha, *Lectures on Probability, Entropy, and Statistical Physics* (MaxEn08, São Paulo, 2008) (arXiv.org/abs/0808.0012).
- [7] C. C. Rodríguez, “The ABC of model selection: AIC, BIC and the new CIC” (omega.albany.edu:8008/CIC/me05.pdf).
- [8] K. H. Knuth, “Lattice duality: The origin of probability and entropy,” Neurocomputing **67C**, 245 (2005).

- [9] A. Caticha, “Entropic Dynamics” in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by R. L. Fry, AIP Conf. Proc. **617**, 302 (2002). ([arXiv.org/abs/gr-qc/0109068](https://arxiv.org/abs/gr-qc/0109068)).
- [10] A. Caticha, “Towards a Statistical Geometrodynamics” in *Decoherence and Entropy in Complex Systems* ed. by H.-T. Elze (Springer Verlag, 2004) ([arXiv.org/abs/gr-qc/0301061](https://arxiv.org/abs/gr-qc/0301061)); “The Information geometry of Space and Time” in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by K. Knuth, A. Abbas, R. Morris, and J. Castle, AIP Conf. Proc. **803**, 355 (2006) ([arXiv.org/abs/gr-qc/0508108](https://arxiv.org/abs/gr-qc/0508108)).
- [11] A. Caticha and C. Cafaro, ‘From Information Geometry to Newtonian Dynamics’, in *Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, ed. by K. Knuth *et al.*, AIP Conf. Proc. **954**, 165 (2007) ([arXiv.org/abs/0710.1071](https://arxiv.org/abs/0710.1071)).
- [12] E. Nelson, “Derivation of the Schroedinger equation from Newtonian Mechanics”, Phys. Rev. **150**, 1079 (1966); *Quantum Fluctuations* (Princeton U. Press, 1985).
- [13] S. Amari and H. Nagaoka, *Methods of Information Geometry* (Am. Math. Soc./Oxford U. Press, Providence, 2000).
- [14] H. Price, *Time’s Arrow and Archimedes’ Point* (Oxford U. Press, 1996); H. D. Zeh, *The Physical Basis of the Direction of Time* (Springer, 2002).